Discussion of Myers-Read

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This study shows how option pricing methods can be used to allocate required capital (surplus) across lines of insurance. The capital allocations depend on the uncertainty about each line's losses and also on correlations with other lines' losses and with asset returns. The allocations depend on the *marginal* contribution of each line to default value—that is, to the present value of the insurance company's option to default. **The authors show that marginal default values add up to the total default value for the company, so that the capital allocations are unique and not arbitrary**. They therefore disagree with prior literature arguing that capital should not be allocated to lines of business or should be allocated uniformly. The study presents several examples based on standard option pricing methods. **However, the "adding up" result justifying unique capital allocations holds for any joint probability distribution of losses and asset returns**. The study concludes with implications for insurance pricing and regulation.

Homogeneity Assumption

For simplicity, we will consider two lines, *a* and *b*, and two dates, $t = 0$ and $t = 1$. Present values (PVs) and outcomes are:

No restriction on joint distribution of R_a and R_b

	PV at $t = 0$	*Outcome* at $t = 1$
Assets	V	$\tilde{V} \equiv V\tilde{R}_V$
Line a	L_a	$\tilde{L}_a \equiv L_a\tilde{R}_a$
Line b	L_b	$\tilde{L}_b \equiv L_b\tilde{R}_b$

A priori mean

Fixed random component

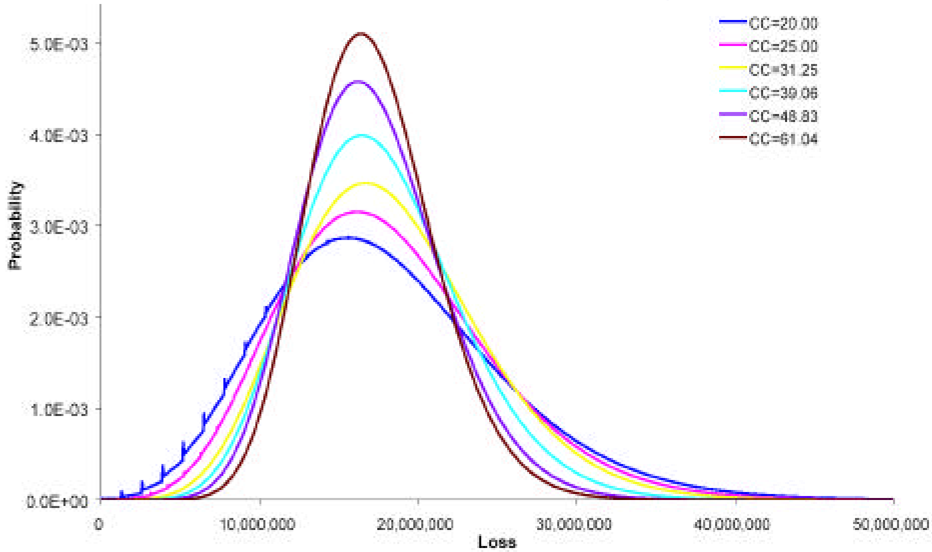
The outcomes are expressed as the product of the initial present values and gross returns $\tilde{R}_V$, $\tilde{R}_a$, and $\tilde{R}_b$. The gross returns are the state variables. Each can be zero but not negative.

Loss random variable is **homogeneous** with respect to expected losses

Stocks are Homogeneous

- Value of a portfolio of ***n*** XYZ Co. stocks is ***n*** times the value of one stock
- Stock values are homogeneous
- Insurance losses are not homogeneous
  - Policies are analogous to individual stocks
- Distribution of losses changes shape as portfolio of policies grows
  - Writing multiple policies on same risk would be homogeneous

Insurance is Not Homogeneous



Conclusion

- Key assumption of MR does not hold for portfolio of insurance risks
- Adding up result does not hold for insurance risks
- Framework still valid and useful
  - See Meyer's discussion

Example

Two lines with normally distributed losses
$L_a \sim N(m_a, s_a^2)$, $L_b \sim N(m_b, s_b^2)$ so $L_a+L_b \sim N(m_a+m_b, s_a^2+s_b^2)$

- If $s_i = c_i m_i^h$
- Losses homogeneous iff h=1
- $m_a=m_b=10$, c=1.0, h=1
- $k_a=0.20m_a$, $k_b=0.30m_b$
- $dD/dm_a=0.1926$
- $dD/dm_b=0.1565$
- $m_a dD/dm_a + m_b dD/dm_b = 3.4909$
- D=3.4909

- h=1.25
- Losses not homogeneous
- $m_a=m_b=10$, c=1.0, h=1.25
- $k_a=0.20m_a$, $k_b=0.30m_b$
- $dD/dm_a = 0.5305$
- $dD/dm_b = 0.4884$
- $m_a dD/dm_a + m_b dD/dm_b = 10.1896$
- D=7.7305

Try yourself: http://www.mynl.com/pptp/mrExample.xls

Counter-Argument 1

"MR proof uses homogeneity but the assumption could be avoided with a more cunning argument"

Rebuttal:

My paper shows adding up holds **if and only if** underlying losses are homogeneous

Counter-Argument 2

"The default option is market valued, the market risk-adjustment introduces homogeneity"

Rebuttal:

Argument does not hold in a world of risk neutral agents (no risk adjustment) and hence result is dependent on risk preferences. No such dependence is noted in the paper. Means are homogeneous.

Counter-Argument 3

"It works for a representative insurer" I.e. for a quota share of the total market

Rebuttal:

True, but misses all dynamics of diversification. Assumption not mentioned in paper.

Are there any "representative insurers" here today?

Counter-Argument 4

"To a first-order approximation, the aggregate distribution does not change shape, so the result is close enough"

Rebuttal:

Expressions with derivatives need a second order approximation; normal distribution example and general theory strongly contradict this argument.